Disturbance Rejection in Flexible Structures via the Quantitative Feedback Theory

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I. Introduction

THE control design problem for flexible structures has been the focus of many investigations in the past few decades. This problem is very interesting from both its theoretical and practical aspects. The possibility of infinite number of modes in the structure dynamics poses a set of difficult mathematical problems; however, in practice, models of flexible structures are uncertain since they are derived from a finite element formulation or an identification procedure. Therefore, in practice, control design for flexible structures must explicitly consider uncertainty. The available literature on both theoretical and practical developments in the area of control of flexible structures is very extensive. Almost every possible control method has been tried on flexible structures with varying degrees of success, either in simulations or in actual experiments. A good source that covers many of these attempts is the book by Meirovitch[1] and the reference therein.

The objective of this Note is to explore the feasibility of using and extending ideas from traditional multi-input/multi-output (MIMO) quantitative feedback theory (QFT) to control of flexible structures. Recent theoretical investigations of control of distributed parameter systems within the framework of QFT can be found in Kelemen et al.[2,3] A single-input/single-output (SISO) QFT control design for a flexible structure was excecuted by Yaniv and Horowitz.[4] To the best of our knowledge, the design procedure for QFT MIMO disturbance rejection is shown here for the first time in detail (a similar procedure was developed for the MIMO tracking problem in Ref. 5). This defines the contribution of this Note. A 2×2 flexible structure example with both parametric and nonparametric uncertainties illustrates the QFT control design procedure.

II. Statement of the Control Problem

Consider a feedback system where the plant, a transfer function matrix (TFM) $P=[p_{ij}]$, includes uncertainties; $P \in \mathcal{P}$, where $\mathcal{P}$ is a family of linear time-invariant TFM. The controller G is placed in the forward loop. The disturbance D is located at the plant input, and the plant output is denoted by Y. All TFM are square with the dimension $n\times n$. The closed-loop system must satisfy the following objectives. For each $P \in \mathcal{P}$, achieve internal stability and disturbance rejection according to

$$Y = TD, \quad T=[t_{ij}]=(I+PG)^{-1}P, \quad |t_{ij}(j\omega)| \le \alpha_{ij}(\omega) \quad (1)$$

The control design problem is to find a diagonal controller G to meet these objectives above for $i,j=1,\dots,n$.

III. Basic Multi-Input/Multi-Output Quantitative Feedback Theory

The assumptions that are typically required for traditional MIMO QFT techniques are: 1) the plant family $\mathcal{P}$ and the controller G have no decentralized "hidden unstable modes"; 2) right-half-plane poles and zeros of the controller G do not coincide with zeros and poles of the family $\mathcal{P}$, respectively; 3) P is square and $\det[P]\neq 0$ for all $P \in \mathcal{P}$; 4) G is diagonal; and 5) the plant family $\mathcal{P}$ can be reasonably approximated with a family of finite members.

The first two assumptions are required for closed-loop internal stability of the QFT procedure and are standard in applications. The third assumption is needed to allow for plant inversion. The fourth assumption greatly simplifies the sequential single-loop design used by QFT. Existence conditions for diagonal controllers within the framework of MIMO QFT are given by Nwokah et al.[6] The last assumption is required because numerical algorithms employed to solve MIMO QFT design problems can only handle finite numbers of plants. Based on these assumptions, QFT should be considered strictly as a control design method for engineering purposes.

IV. Design Procedure

Stability

Closed-loop robust stability is arrived at using a similar idea which first appeared in Ref. 7. It is based on a sequential SISO design that incorporates the generalized inverse Nyquist stability criterion. The purpose of using a sequential procedure is to arrive at a final form $I+(PG)^{-1}=LU$, where $\det[L]=1$ and $U=[u_{ij}]$ is an upper triangular TFM. Having obtained this form we can take advantage of $\det[I+(PG)^{-1}]=\det[U]=\Pi u_{ii}$. This form can be arrived at as follows. Write out $I+(PG)^{-1}$ in full form

$$\begin{bmatrix} 1+q_{11}/g_1 & q_{12}/g_1 & \cdot & q_{1n}/g_1 \\ q_{21}/g_2 & 1+q_{22}/g_2 & \cdot & q_{2n}/g_2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ q_{n1}/g_n & \cdot & \cdot & 1+q_{nn}/g_n \end{bmatrix}$$

where for simplicity we use the notation $Q=[q_{ij}]=P^{-1}$. Without loss of generality we assume that the design proceeds in order from the first row to the last row. For the first row, except for pathological cases, we can always find a controller g_1 such that $1+q_{11}/g_1\neq 0$ for each $P \in \mathcal{P}$, since the family $\mathcal{P}$ has only a finite number of elements. The details related to design of g_1 to meet closed-loop objectives are given later in this section. Once g_1 is designed, we proceed to find the controller g_2 for the second row. However, because g_1 is known, we can imbed it into the second row using the Gauss elimination method, as suggested by Yaniv and Horowitz.[7] In fact, this trick has already been suggested in the context of fixed systems by Mayne.[8,9] The elimination is performed by subtracting from the second row the quantity $(q_{21}/g_2)/(1+q_{11}/g_1)$ times the first row. At this stage we can proceed to the next row, i.e., design g_2 such that $1+q_{22}^2/g_2\neq 0$ for each $P \in \mathcal{P}$. Next, we imbed the known controllers, g_1 and g_2, into the third row using Gauss elimination and solve for g_3. This design process of a single g_i and subsequent elimination is repeated $n-1$ times where the general formula for Gauss elimination is

$$q_{mj}^m = q_{mj}^1 - \sum_{i=1}^{m-1} \frac{q_{mi}^i q_{ij}^i}{(g_i+q_{ii}^i)}, \qquad m=2,3,\dots,n \quad (2)$$

where $q_{ij}^1=q_{ij}$. The final result takes the TFM $I+(PG)^{-1}$ into the product LU, where L is unimodular lower triangular matrix and U is upper triangular TFM. This is otherwise known as the Gauss elimination[10] method. In general, an LU decomposition has the form $P_1[I+(PG)^{-1}]=LU$, where P_1 is a permutation matrix that reorders the rows of $I+(PG)^{-1}$ so that $P_1[I+(PG)^{-1}]$ admits a factorization with nonzero pivots (this is equivalent to reordering of inputs and outputs). In

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the QFT procedure the pivots in $I+(PG)^{-1}$ are made nonzero by design, so we have $I+(PG)^{-1}=LU$, or in full form

$$\begin{bmatrix} 1+q_{11}^1/g_1 & q_{12}^1/g_1 & \cdot & q_{1n}^1/g_1 \\ q_{21}/g_2 & 1+q_{22}/g_2 & \cdot & q_{2n}/g_2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ q_{n1}/g_n & \cdot & \cdot & 1+q_{nn}/g_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ * & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ * & \cdot & \cdot & \cdot & * & 1 & 0 \\ * & \cdot & \cdot & \cdot & \cdot & * & 1 \end{bmatrix} \begin{bmatrix} 1+q_{11}^1/g_1 & q_{12}^1/g_1 & \cdot & \cdot & q_{1n}^1/g_1 \\ 0 & 1+q_{22}^2/g_2 & \cdot & \cdot & q_{2n}^2/g_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & 1+q_{(n-1)(n-1)}^{n-1}/g_{n-1} & q_{(n-1)n}^{n-1}/g_{n-1} \\ 0 & \cdot & \cdot & 0 & 1+q_{nn}^n/g_n \end{bmatrix}$$

where the asterisk denotes a complex coefficient. Note that $\det[L]=1$ is invariant to the family $\mathcal{P}$.

As mentioned earlier, the sequential MIMO QFT technique arrives at a robust internally stable system by employing a criterion similar to the generalized inverse Nyquist stability criterion as follows. Define the Nyquist plot of a function as its image under the usual Nyquist contour with left indentations. Denote the right-half s-plane zeros of PG by n_z and assume that n_z is the same integer for each $P \in \mathcal{P}$.

Theorem: Under Assumptions 1-5 in Sec. III, the QFT design is robust internally stable if, and only if, for each $P \in \mathcal{P}$, the net sum of counterclockwise encirclements of $(-1,0)$ by the inverse characteristic loci of

$$\det\left[I+(PG)^{-1}\right] = \prod_{i=1}^{n}(1+q_{ii}^i/g_i)$$

is equal to n_z.

The proof of this theorem[11,12] is a straightforward extension of results in Rosenbrock,[13] Desoer and Chan,[14] Postlethwaite,[15] and Anderson and Gevers.[16] At each design step one can apply traditional SISO QFT.[17]

Disturbance Rejection

The control design here is very similar to the sequential design for robust stability. Indeed, the disturbance TFM $T=(I+PG)^{-1}P$ can be written as $[I+(PG)^{-1}]T=G^{-1}$. In a full matrix form it looks like

$$\begin{bmatrix} 1+q_{11}^1/g_1 & q_{12}^1/g_1 & \cdot & q_{1n}^1/g_1 \\ q_{21}/g_2 & 1+q_{22}/g_2 & \cdot & q_{2n}/g_2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ q_{n1}/g_n & \cdot & \cdot & 1+q_{nn}/g_n \end{bmatrix} \begin{bmatrix} t_{11} & \cdot & t_{1n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ t_{n1} & \cdot & t_{nn} \end{bmatrix} = \begin{bmatrix} 1/g_1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1/g_2 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & 1/g_n \end{bmatrix}$$

The sequential design proceeds as shown earlier; a single controller is found based on the equations for a specific row. Subsequently, using Gauss elimination the design proceeds in a sequential manner until we arrive at the last row

$$\begin{bmatrix} 1+q_{11}^1/g_1 & q_{12}^1/g_1 & \cdot & \cdot & q_{1n}^1/g_1 \\ 0 & 1+q_{22}^2/g_2 & \cdot & \cdot & q_{2n}^2/g_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & 1+q_{(n-1)(n-1)}^{n-1}/g_{n-1} & q_{(n-1)n}^{n-1}/g_{n-1} \\ 0 & \cdot & \cdot & 0 & 1+q_{nn}^n/g_n \end{bmatrix} \begin{bmatrix} t_{11} & \cdot & t_{1n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ t_{n1} & \cdot & t_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} c_{11}/g_1 & 0 & 0 & \cdot & \cdot & 0 \\ c_{21}/g_2 & c_{22}/g_2 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{(n-1)1}/g_{n-1} & \cdot & \cdot & c_{(n-1)(n-2)}/g_{n-1} & c_{(n-1)(n-1)}/g_{n-1} & 0 \\ c_{n1}/g_n & \cdot & \cdot & \cdot & c_{n(n-1)}/g_n & c_{nn}/g_n \end{bmatrix} \tag{3}$$

The general elimination formulas are

$$q_{mj}^m = q_{mj}^1 - \sum_{i=1}^{m-1} \frac{q_{mi}^i q_{ij}^i}{g_i + q_{ii}^i}, \qquad m = 2,3,\ldots,n$$

$$c_{mj} = -\sum_{i=1}^{m-1} \frac{q_{mi}^i c_{ij}}{g_i + q_{ii}^i}, \qquad m = 2,3,\ldots,n, \quad j = 2,3,\ldots,n-1$$

where $c_{mj}=1$ if $m=j$ (note that $c_{ij}=0$ if $i<j$).

From Eq. (3) we obtain a set of n equations at each design step which can be rearranged to isolate the closed-loop transfer functions at that loop. At each design step, these are the final relations for each t_{ij}, $i,j = 1,\ldots,n$,

$$|t_{ij}| = \left| \frac{c_{ij} - \sum_{k=i+1}^{n} q_{ik}^{i} t_{kj}}{g_i + q_{ii}^{i}} \right| \leq \alpha_{ij}(\omega) \tag{4}$$

Note that at each ith step we do not have exact knowledge of t_{kj}, $k = i+1,\ldots,n$, $j = 1,\ldots,n$. Therefore, the set of inequalities (4) cannot be solved unless we employ some type of assumptions on these t_{kj}. For this purpose assume that there exists a linear, diagonal, time invariant controller G which solves the problem; that is, $|t_{kj}(j\omega)| \leq \alpha_{kj}(\omega)$, $k = 1,\ldots,n$, $j = 1,\ldots,n$. For example, this assumption allows the following conservative simplification in the first design step; let

$$\delta_1(\omega) = 1 + \left| \sum_{j=2}^{n} |q_{1j}^{1}|\, \alpha_{j1} \right|$$

$$\delta_j(\omega) = \left| \sum_{j=2}^{n} |q_{1j}^{1}|\, \alpha_{j1} \right|, \qquad j = 2,\ldots,n \tag{5}$$

then plug into the relations for $|t_{ij}|$ in Eq. (4) to obtain

$$\left| \frac{1}{g_1 + q_{11}^{1}} \right| \leq \frac{\alpha_{11}(\omega)}{\delta_1(\omega)} = \delta_{11}(\omega)$$

$$\left| \frac{1}{g_1 + q_{11}^{1}} \right| \leq \frac{\alpha_{1j}(\omega)}{\delta_j(\omega)} = \delta_{1j}(\omega), \qquad j = 2,\ldots,n \tag{6}$$

Note that if the inequalities (6) are met, then it is implied that the disturbance rejection objective (4) is also met but not the other way around. This is where MIMO QFT adds conservatism into the design. The preceding inequalities can be transformed to quadratic inequalities whose solutions are easily obtained. Specifically, consider the first inequality in Eq. (6) and let $g_1(j\omega) = ge^{j\phi}$ and $q_{11}^{1}(j\omega) = qe^{j\theta}$. Fix the frequency, plug these into the inequality, square both sides, and simplify in terms of the gain g

$$\frac{g^2}{q^2} + 2\frac{g}{q}\cos(\theta - \phi) + 1 - \frac{1}{q^2\delta^2} \geq 0 \tag{7}$$

where $\delta(\omega) = \min\{\delta_{1j}(\omega)\}$, $j = 1, 2, \ldots, n$. Given this set of quadratic inequalities (one for each $P \in \mathcal{P}$), QFT bounds are computed followed by loop shaping as described in Ref. 18.

V. Example

The simple plant used in this example includes both of the parametric and nonparametric uncertainties to highlight the possible use of our QFT design procedure to the general class of problems of disturbance rejection in flexible structures. The uncertain plant is described by $P = [p_{ij}]$, $i,j = 1,2$, where $p_{11} = a_1/(s^2 + 0.4s + 10)$, $a_1 \in [6,8]$; $p_{12} = 3/(s^2 + b_1 s + 10)$, $b_1 \in [0.2,0.4]$; $p_{21} = 0.2/(s^2 + 0.4s + 10)$; and $|p_{22} - p_{220}| \leq 0.001(1 + \omega/10)$, $p_{220} = 8/(s^2 + 0.4s + 10)$, p_{22} stable. Note that p_{11} and p_{12} have parametric uncertainties, p_{21} has no uncertainty, and p_{22} has a nonparametric uncertainty. The disturbance rejecting weights are $\alpha_{1j}(\omega) = 0.1(\omega + 1)$ and $\alpha_{2j}(\omega) = 0.5(\omega + 1)$, $j = 1,2$. Note that the desired rejection is relaxed in the second channel as a direct consequence of the unstructured uncertainty in p_{22}. Specifically, at approximately $\omega = 40$ the phase of the uncertain p_{22} is arbitrary (i.e., anywhere in [0 deg, 360 deg]). This implies that if strictly proper loop transmissions are to be used, the useful bandwidth in the second loop must be well below this frequency.

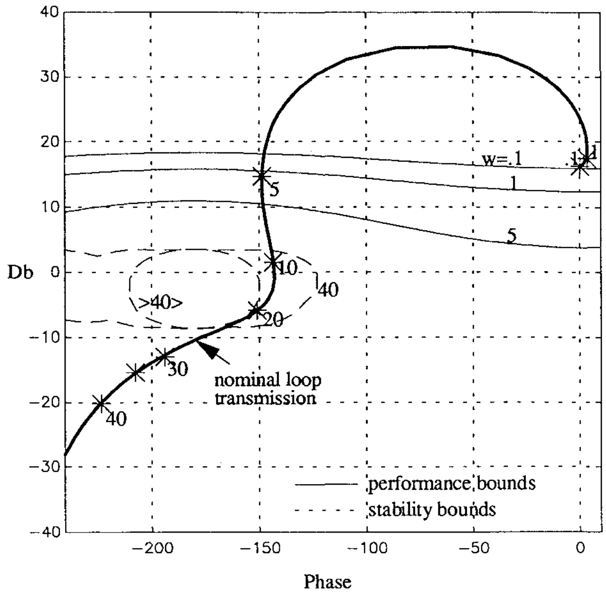

Fig. 1 QFT bounds and nominal design in the first loop.

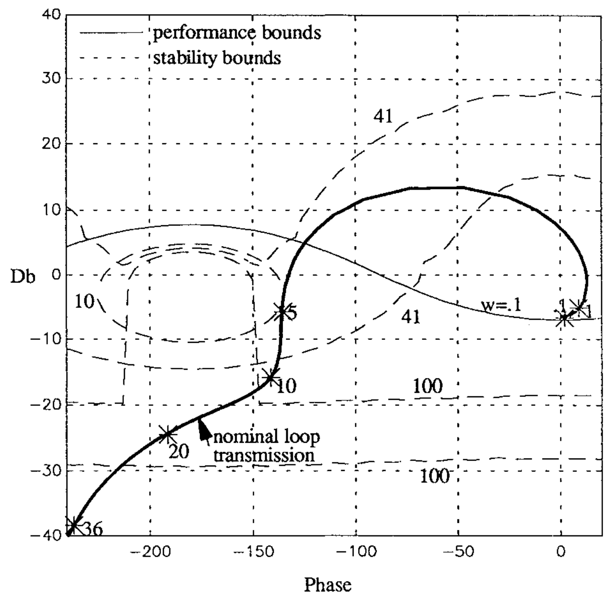

Fig. 2 QFT bounds and nominal design in the second loop.

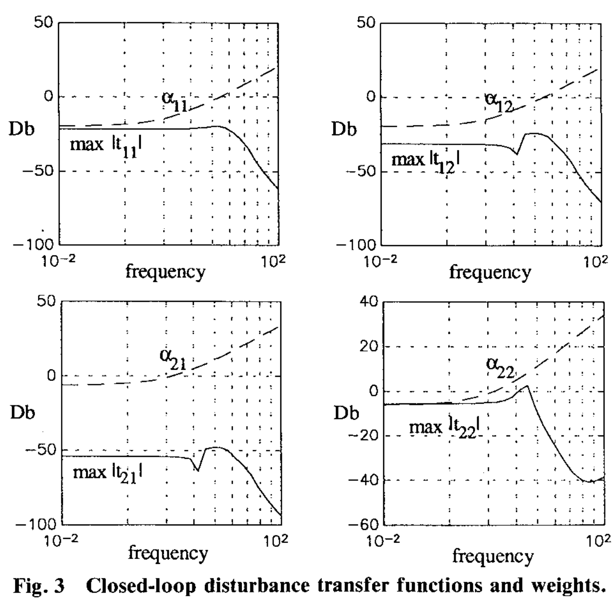

Fig. 3 Closed-loop disturbance transfer functions and weights.

An in-house MIMO QFT program was used to solve this example. The first design step in this example was arbitrarily chosen to consider the first row. The nominal plant was chosen at $a_1 = 6$, $b_1 = 0.2$ and p_{220}. The computed solution for Eq. (7), i.e., the QFT bounds on the nominal $L_{10} = g_1/q_{110}$ are shown in Fig. 1. The compensator is described by

$$g_1 = 11\,\frac{s/8+1}{s^2/35^2+s/35+1}$$

As described in Sec. IV, after g_1 was designed, we applied Gauss elimination to imbed it into the second loop. The bounds for the nominal loop $L_{20} = g_2/q_{220}^2$ were computed using equations which are similar to Eq. (7). The compensator for the solution shown in Fig. 2 is described by

$$g_2 = 0.6\,\frac{s/4+1}{s^2/20^2+s/20+1}$$

The maximum magnitudes of the disturbance transfer functions are compared to their corresponding weights in Fig. 3.

VI. Conclusions

A control design procedure for disturbance rejection in flexible structures with uncertain models was developed based on ideas from traditional multi-input/multi-output quantitative feedback theory. The models can include uncertainties in both parametric and nonparametric forms. It was shown that computation of quantitative feedback theory bounds can be simplified into an n-step sequential design procedure. At each ith step, the ith element of the diagonal controller is computed from n quadratic inequalities. A 2×2 flexible structure example was used to illustrate the technique.

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Robust Control of Flexible Structures Using Residual Mode Filters

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Introduction

THE design of stabilizing controllers for infinite-dimensional systems such as aircraft, large flexible space structures, and heat flow is often difficult. For the flexible structure control, a compensator design involves maintaining closed-loop stability in the face of the spillover phenomenon.[1] For such a design to be successful, the stability of the resulting control system needs to be robust to the modeling inaccuracies. Recently, the problem of spillover suppression to avoid instability has been suggested in the literature using the residual mode filter (RMF).[1-5] This Note intends to develop new robust stability criteria for this problem with considerations of the cascaded and parallel RMF-based controller designs. It will become clear how the additional RMF can effectively reduce the spillover instability from a modern robust control viewpoint.

Notation

The induced norm $\|E(j\omega)\| = \sqrt{\lambda_{\max}[E^H(j\omega)E(j\omega)]}$ where $\lambda_{\max}$ denotes the maximum eigenvalue and the superscript H denotes the complex conjugate transpose. If $E(s) \in \mathcal{S}^{m\times n}$, with $\mathcal{S}^{m\times n}$ denoting the set of $m \times n$ matrices whose elements are proper stable rational functions, then

$$\|E(j\omega)\|_\infty = \sup_{\omega\geq 0}\|E(j\omega)\|$$

Define $\mathcal{R}_+ \equiv \{r \in \mathcal{R}: r \geq 0\}$, $\mathcal{C}_- \equiv \{s \in \mathcal{C}: Re(s)<0\}$, and $\mathcal{C}_+ \equiv \mathcal{C} - \mathcal{C}_-$. Also, det(·) indicates the matrix determinant and $\langle\cdot,\cdot\rangle$ denotes the inner product on the Hilbert space $\mathcal{H}$.

Flexible System Model

The dynamic behavior of a flexible structure is given by the linear evolution equation:

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = x_0 \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

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